Exercise Set #8

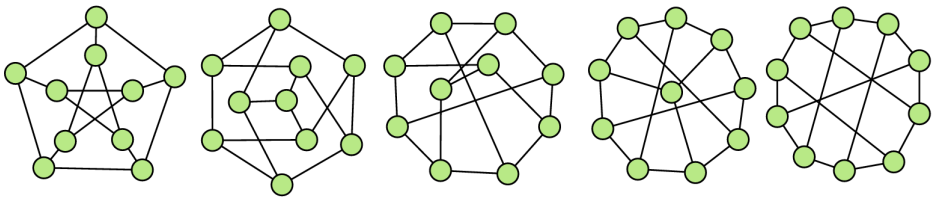
"Discrete Mathematics" (2025)

Exercise 8 is to be submitted on Moodle before 23:59 on April 14th, 2025

E1. **(a)** Is there a graph with 6 vertices with degrees $2, 3, 3, 3, 3, 3$?

(b) How many labeled graphs are there with 4 vertices with degrees $1, 1, 2, 2$?

E2. Are all the graphs below isomorphic to each other? Which ones are/are not? Why?
Hint: Try to move around the vertices of the graphs using a pencil and an eraser, or on a whiteboard.



E3. Let $G = (V, E)$ be a graph and let $|V| = n$.

(a) What is the maximum number of automorphisms (isomorphisms from G to G) possible? Can you find a graph G with this number of automorphisms?

(b) What is the least number of automorphism possible? Can you find a graph with those many automorphisms?

E4. Find the trees that have the following Prüfer sequences:

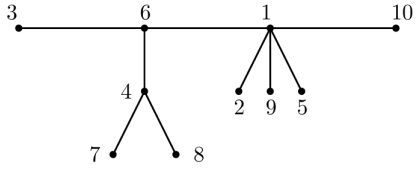
$$(4, 4, 3, 1, 1); (4, 2, 1, 1, 3)$$

E5. **(a)** Describe which Prüfer codes correspond to stars (i.e. to trees where one vertex is connected to all other vertices)

(b) Describe what trees correspond to Prüfer codes containing exactly 2 different values.

(c) And which trees have all distinct values in their Prüfer codes?

E6. What is the Prüfer code of the following tree?



E7. Let v be a vertex of a labeled tree T. Suppose v has degree d, then show that the label of v appears in the Prüfer code of T exactly $d-1$ times.

E8. (**Exercise to submit**)
If C is a cycle, and e is an edge connecting two nonadjacent nodes of C, then we call e a *chord* of C. Prove that if every node of a graph G has degree at least 3, then G contains a cycle with a chord.